Experiments with toy models (ongoing project)

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Summary

- Information Theory for measuring Complex Dynamics
- Input Entropy - Mutual Information - Difference Pattern Spreading Rate
- Toy models experiments
  - Simulation Run Example!
  - Input Entropy / Standard Deviation
  - Mutual Information
  - Difference Pattern Spreading Rate.
- Generalization

Our Experiments With CAs

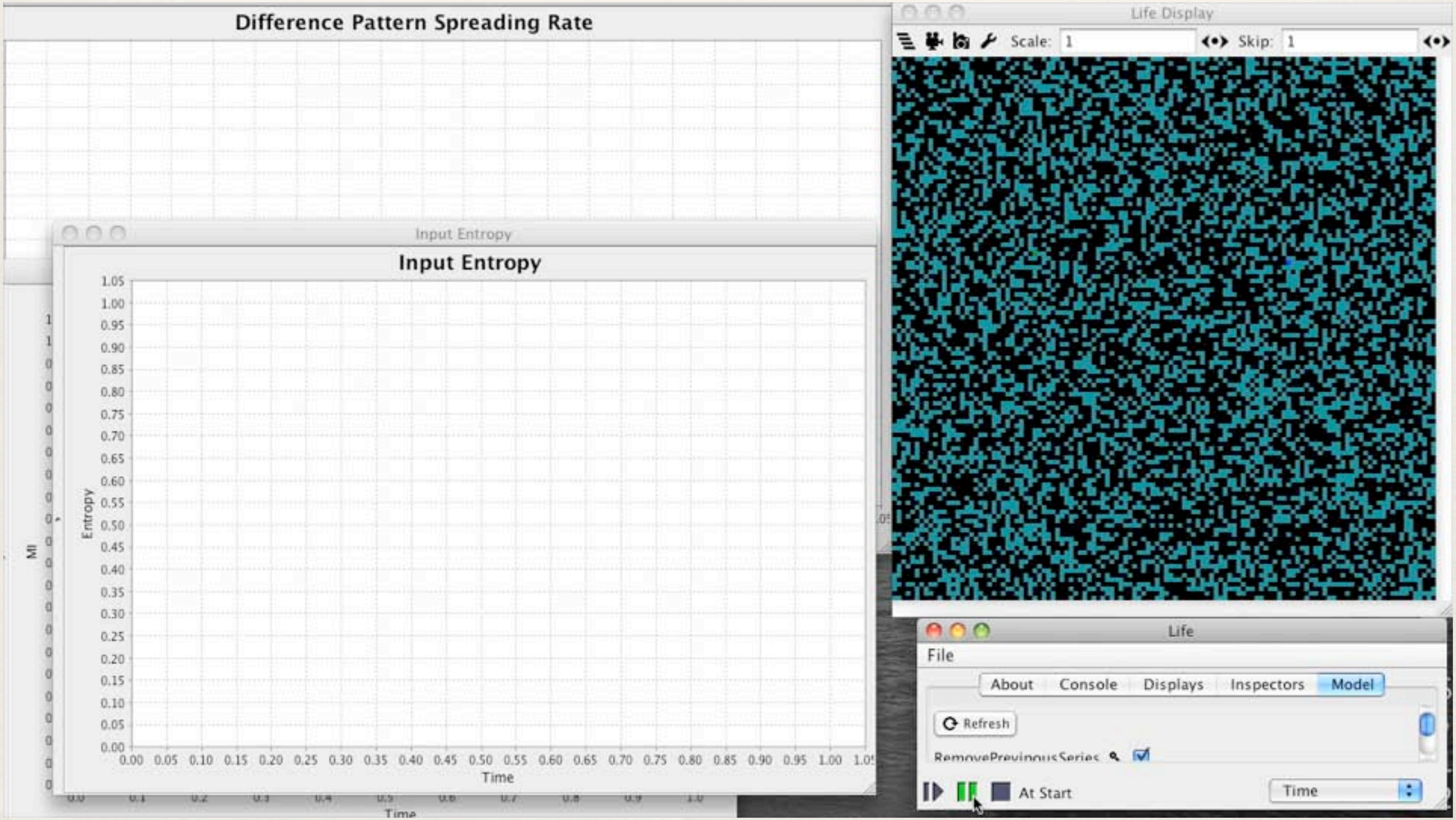
- **Objective**: Explore the limits and implications of some practices in the detection of Complex Behavior and Emergence

- Wolfram Rule 110 (and similar)
- Conway's Life (B3S23) (and similar)
  - Input Entropy / Variance
  - Mutual Information between Consecutive States
  - Difference Pattern Spreading Rate

Experimental Setup

- Life board of size 120x120 in a torus configuration for boundary conditions
- Initial Random Distribution of Lit Cells (quiescent state), from 5% to 95% (19 data points)
- Run the simulation 50 times for averaging purposes for each data point over 2500 timesteps
- Headless deployment in a parallel Grid Setup.

Video Demo of the Setup



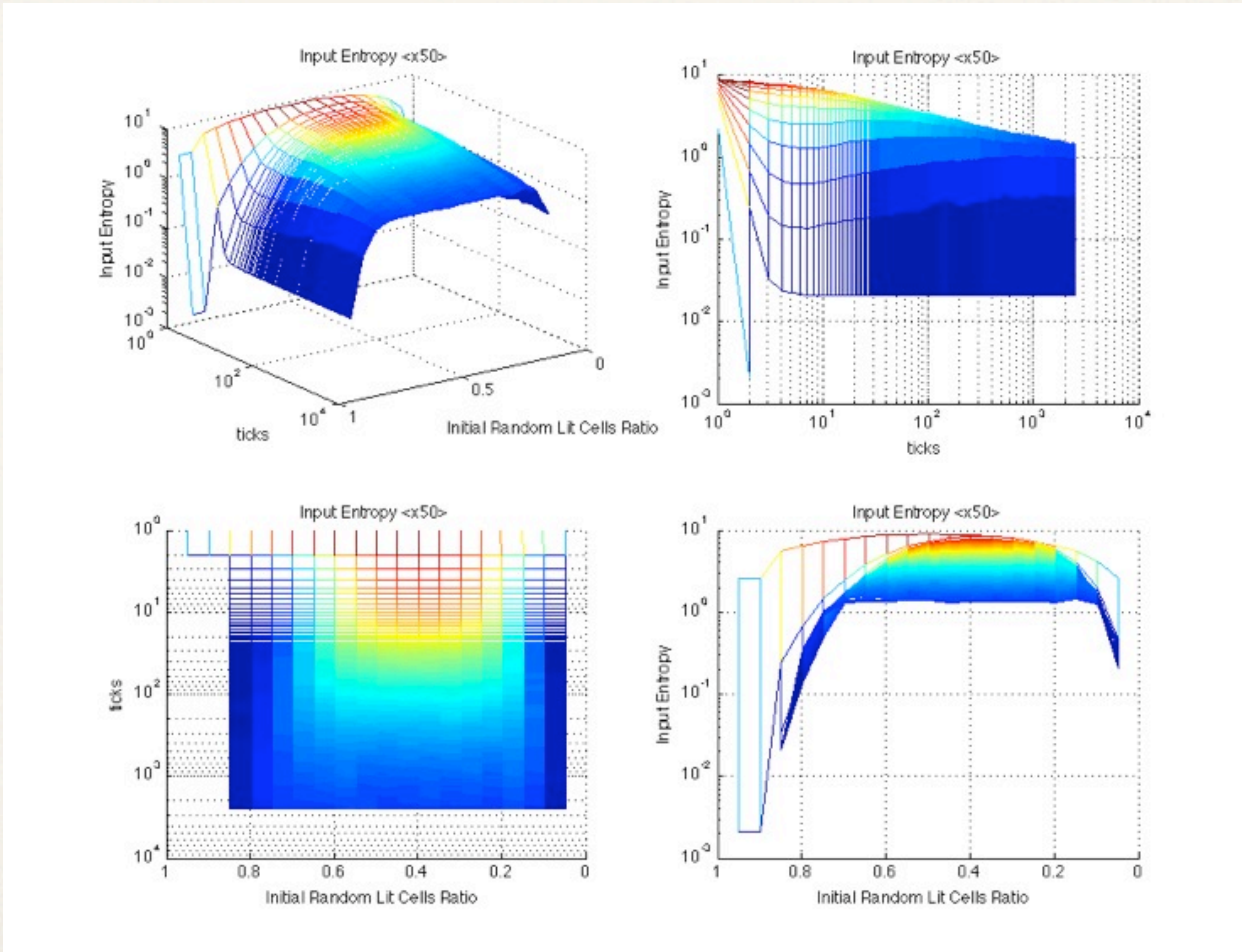
Input Entropy

- Accounts for the **uncertainty/variability** present in the histogram of transition function Δ that is applied at each timestep.

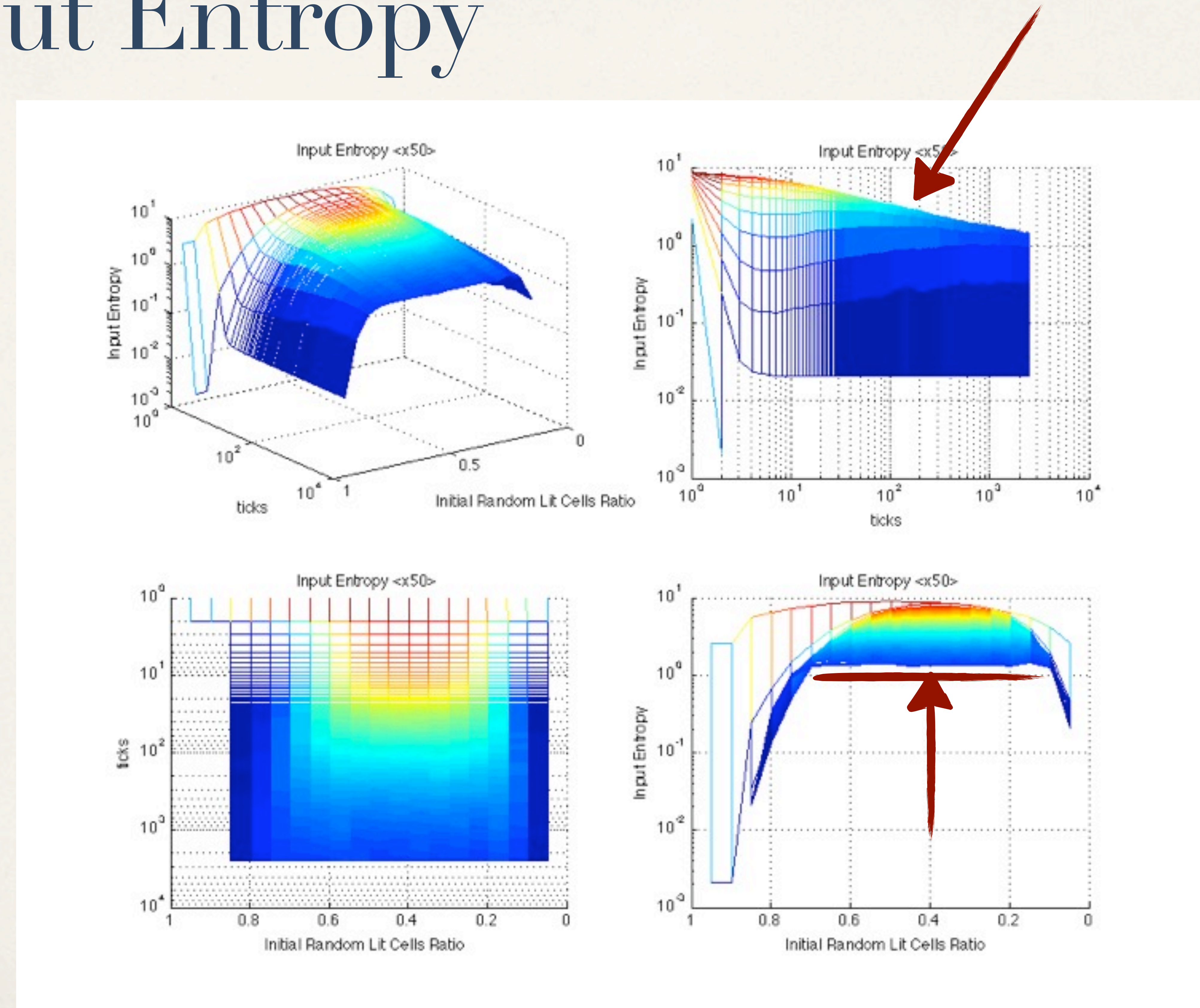
$$S^t = -\sum_{i=1} \left(\frac{Q_i^t}{n} \times \log\left(\frac{Q_i^t}{n}\right) \right)$$

- High Values of Input Entropy => Interesting Dynamics (complex?)
- Maximal Standard Deviation Zone => Interesting Dynamics.

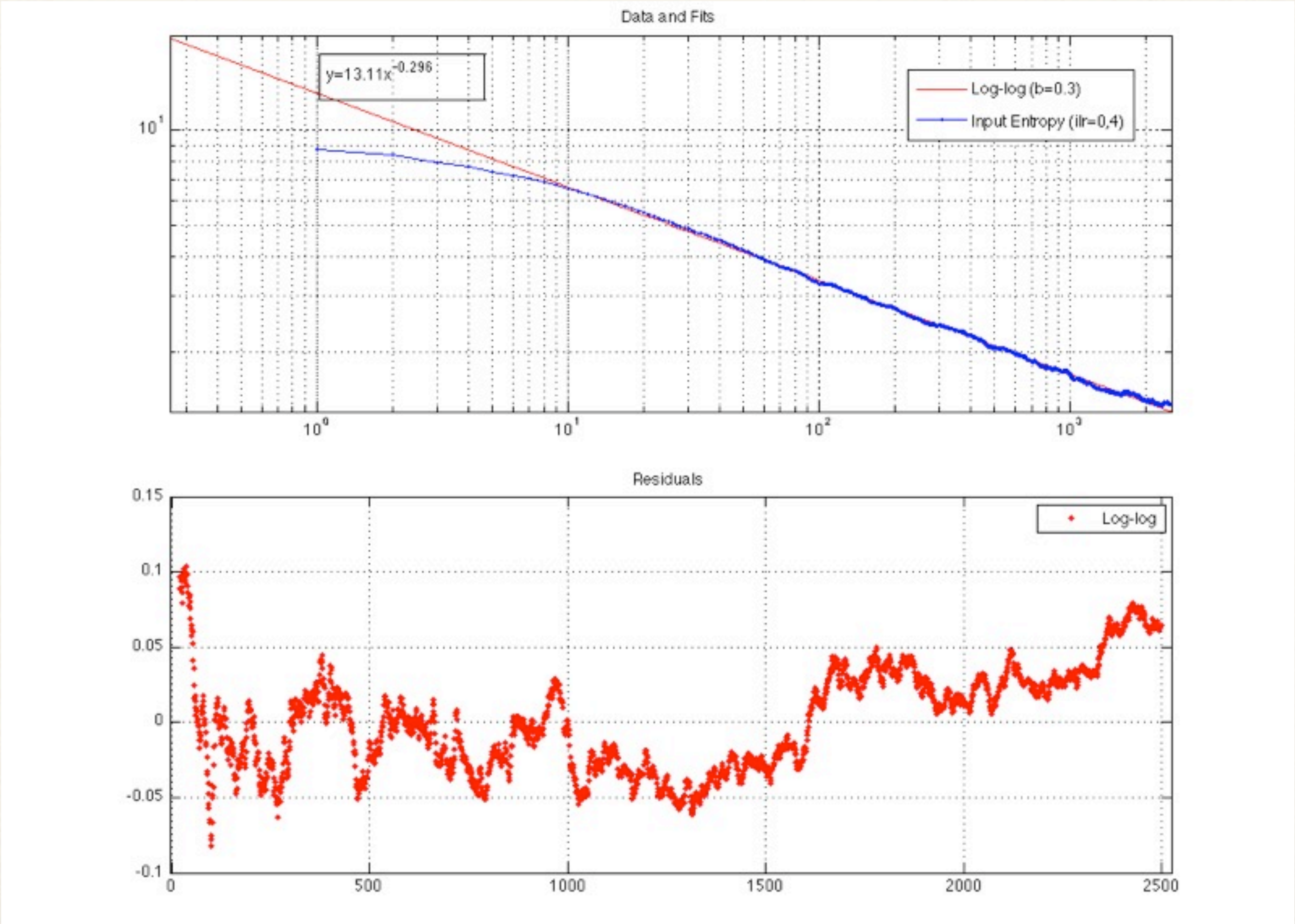
Input Entropy



Input Entropy

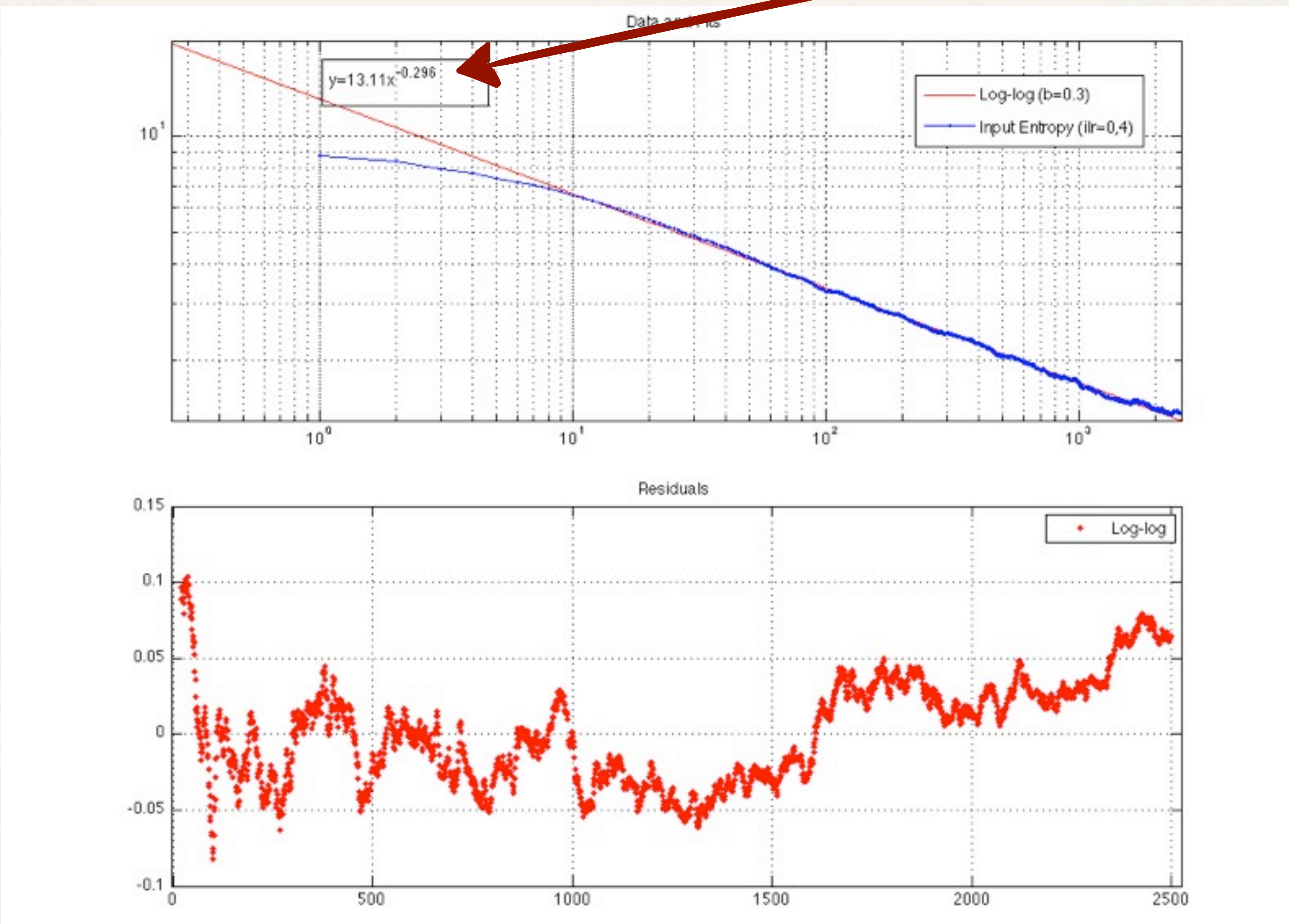


Input Entropy decay

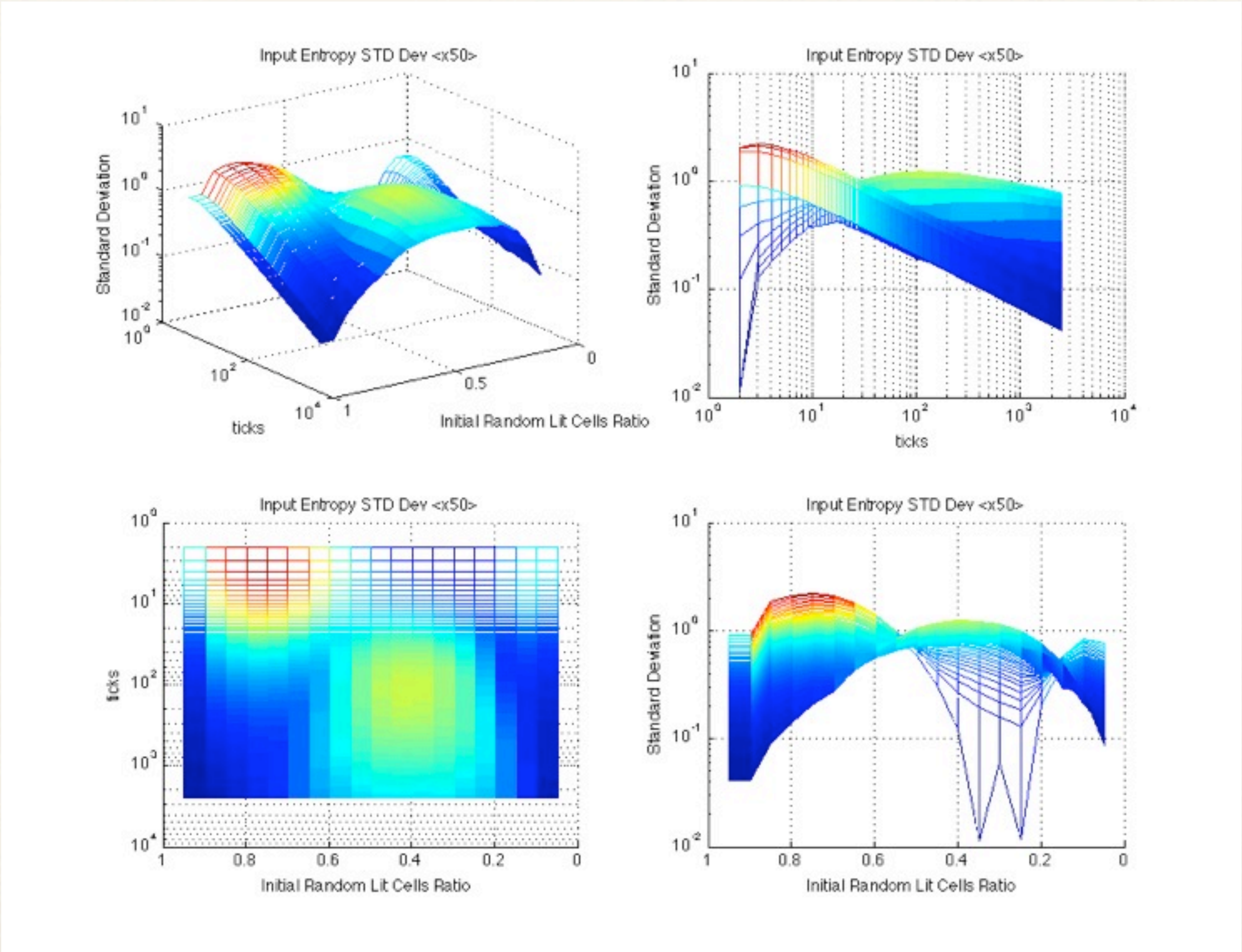


Input Entropy decay

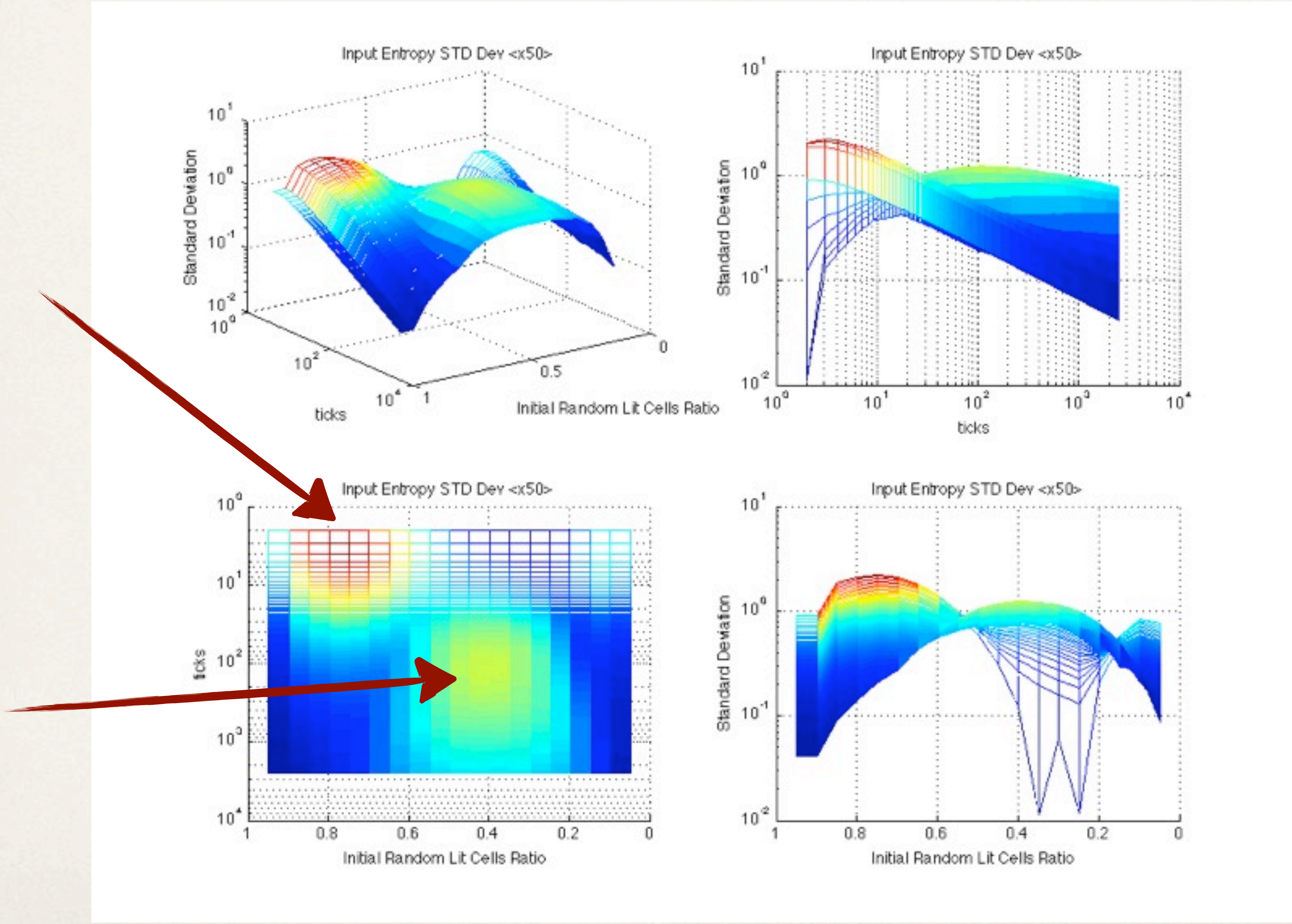
≈-0,3



Input Entropy Standard Deviation



Input Entropy Standard Deviation



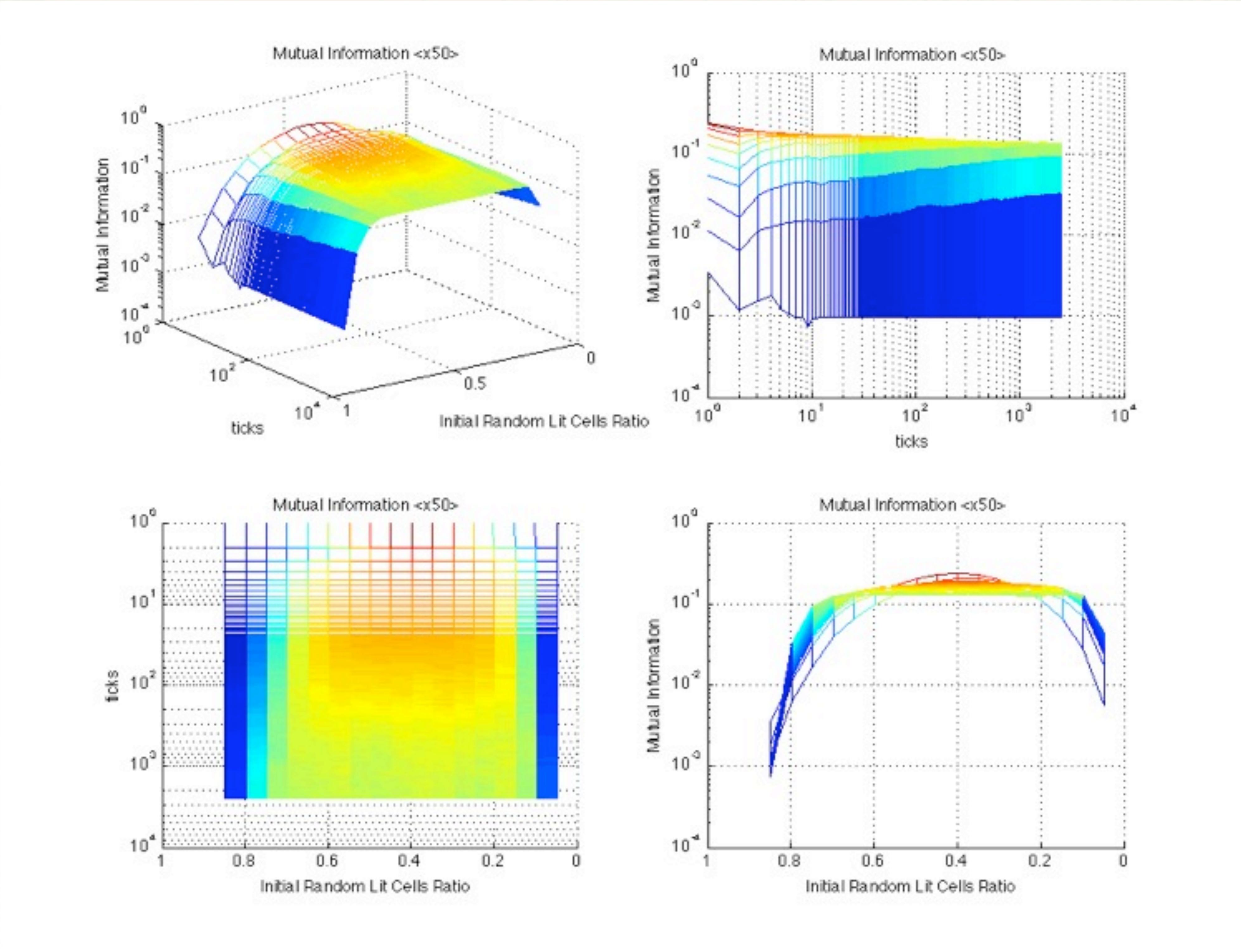
Mutual Information of consecutive states

- Accounts for the information that a state as that is due to the other state

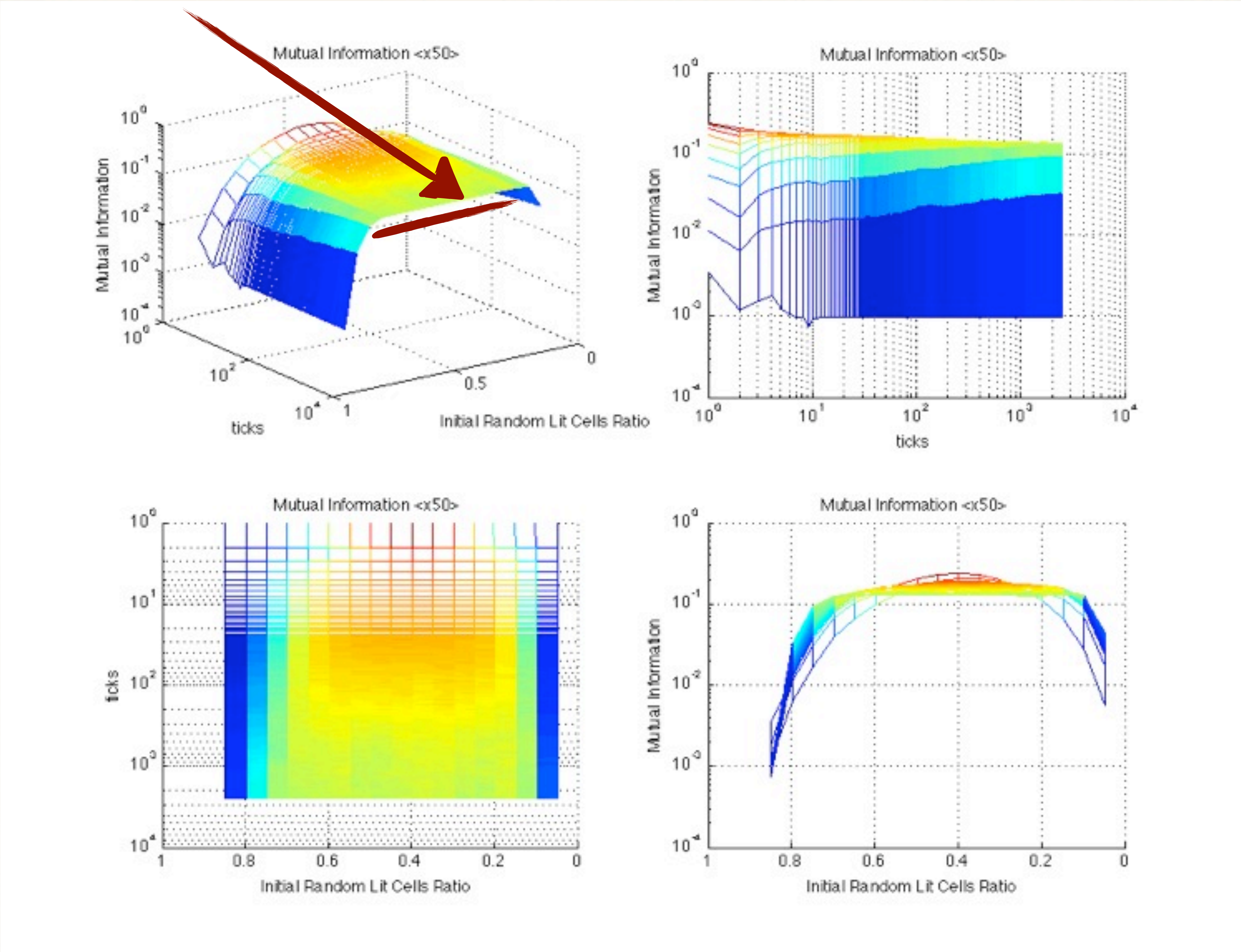
$$I(X;Y) = H(X) - H(X \mid Y) = \sum_{k=1}^{m} \sum_{k=1}^{m} P_{X,Y} \log\left(\frac{P_{X,Y}}{P_X P_Y}\right)$$

- Consecutive states (t->t+1)
- High Values of Mutual Information => Dependence of one state from the other
- Maximal Zone => Interesting Dynamics (More information is carried through the simulation)

Mutual Information



Mutual Information



Difference Pattern Spreading Rate

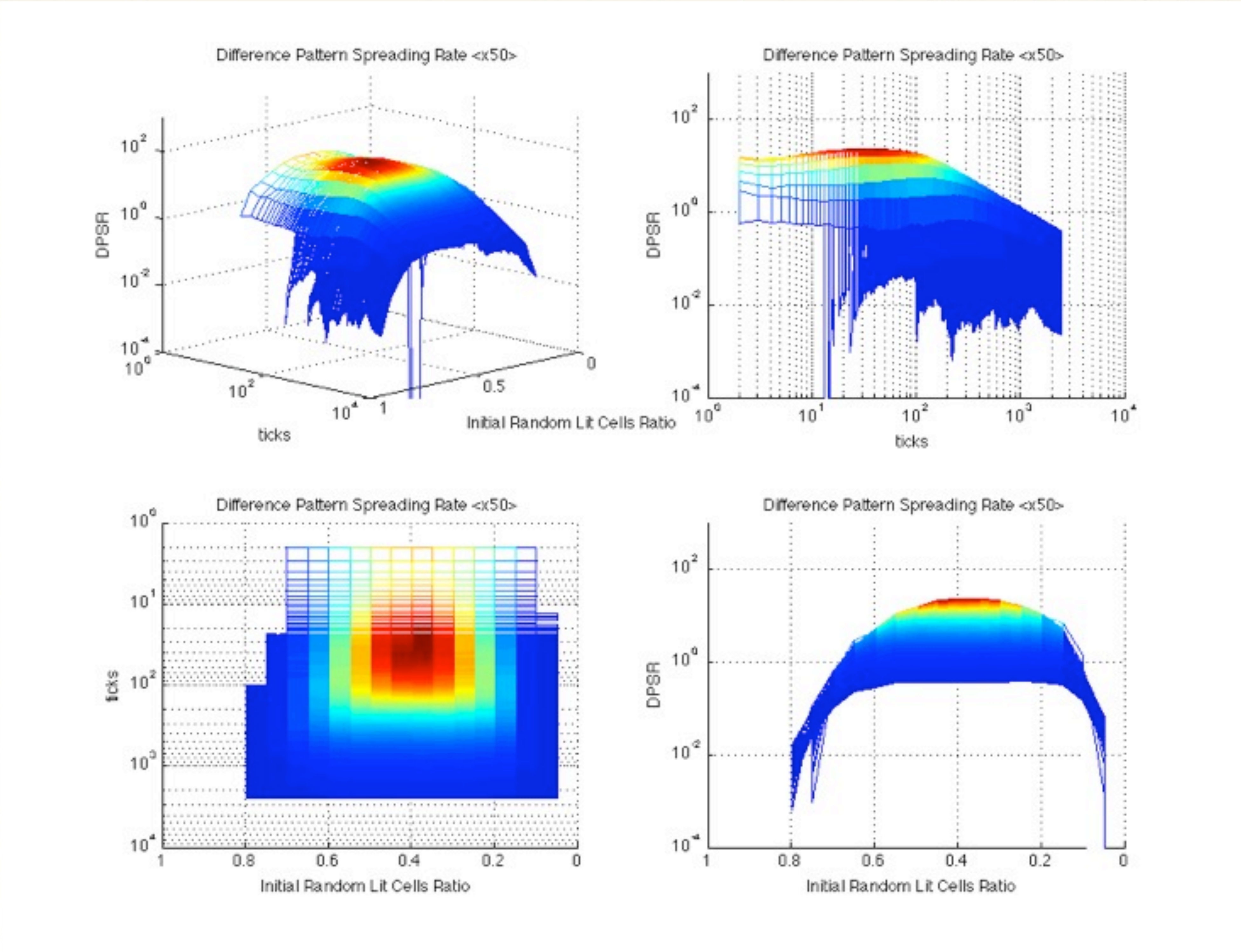
- Measures the rate of "divergence" of two runs when only a small fraction of initial automata start in different states.

$$d(X(t),Y(t)) = \sum_{i,j} \left| X_{ij}(t) - Y_{ij}(t) \right|$$

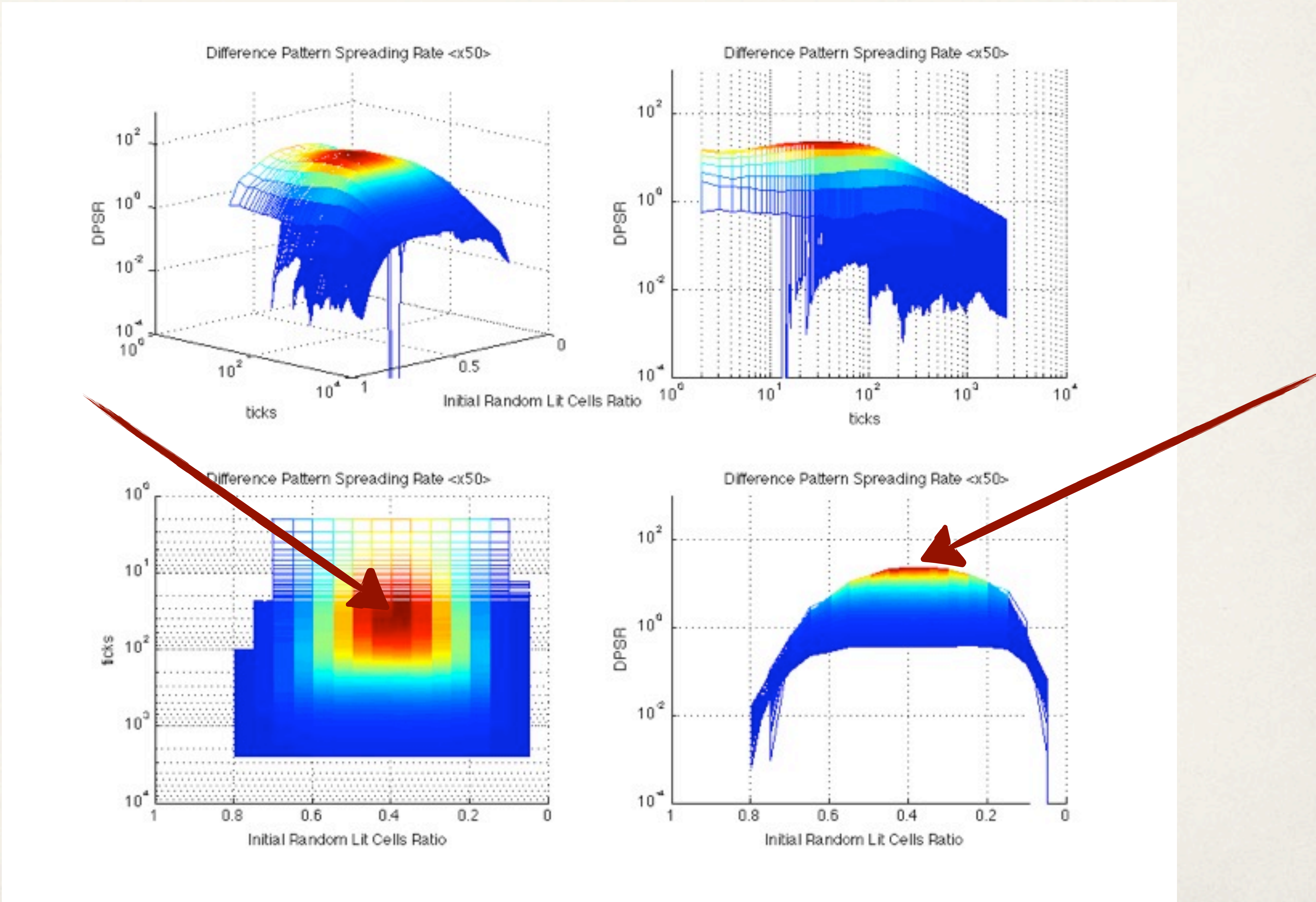
$$\gamma = \frac{d(X(t),Y(t)) - d(X(t_0),Y(t_0))}{t - t_0}$$

- Accounts for the dependency of initial conditions.
- High Values of DPSR => 2 Runs diverge at the maximal rate...
- Maximal Zone => Interesting Dynamics

Difference Pattern Spreading Rate



Difference Pattern Spreading Rate



Classify CA behavior

- 1. Stable configurations. These include oscillators and homogeneous configurations as when all cells die.
- 2. Complex Dynamics. The system presents high values of Input Entropy and Variance, Mutual Information of consecutive states and in Difference Pattern Spreading Rate.
- 3. The system is undistinguishable from a random system generator in the timeframe of analysis.

Type 2 - Complex Dynamics.

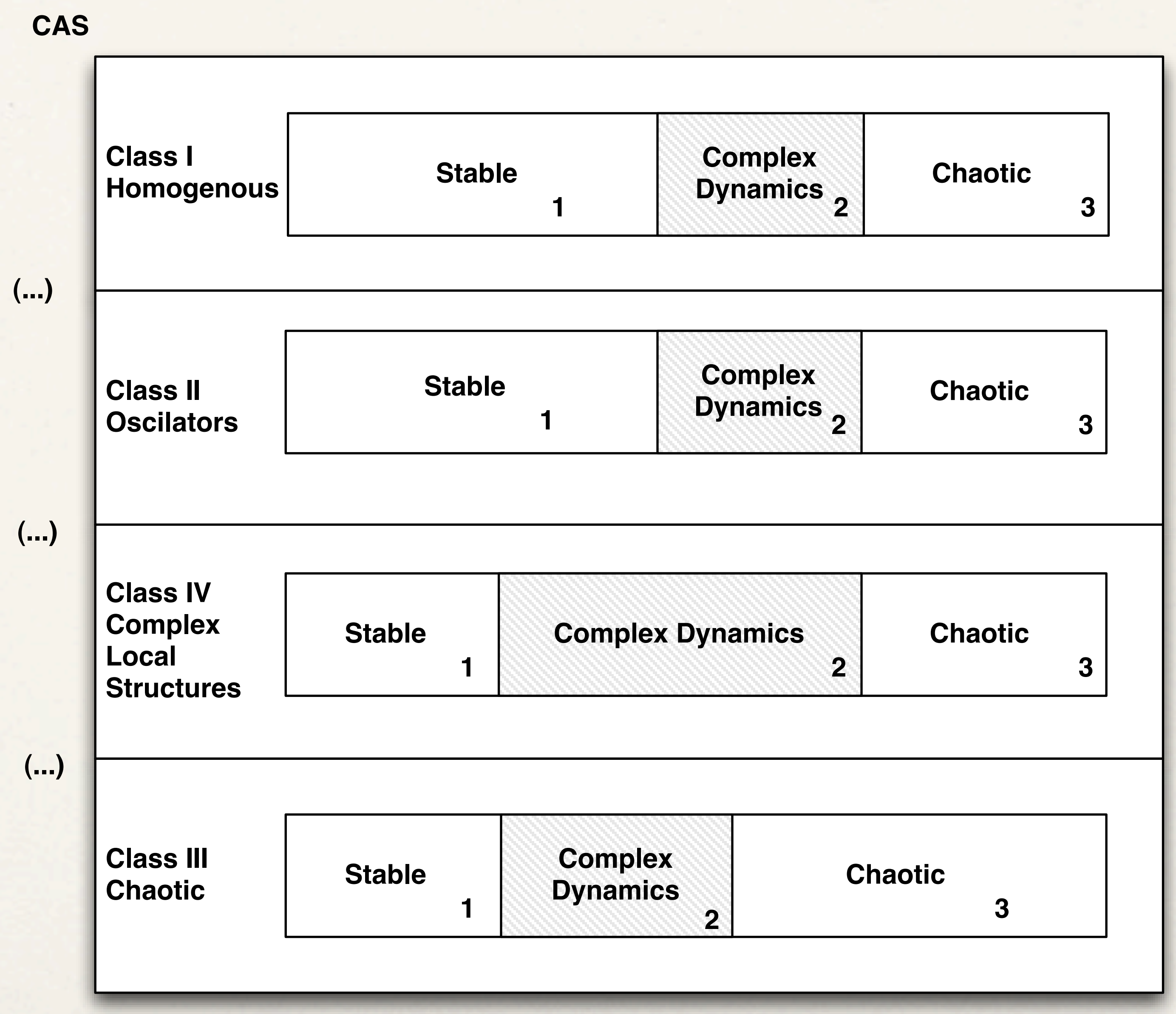
- Have Both High values of Input Entropy and Standard Deviation
- The DPSR test will reveal high values of DPSR
- Mutual Information is carried along the simulation

- Combination of these properties.

Wolfram Class Classification?

- Is this classification similar to Wolframs (1984) for Sets of CAs?
  - Class I - Homogenous state
  - Class II - Simple periodic structures
  - Class III - Chaotic aperiodic patterns
  - Class IV - Complex patterns of localized structures

Multi-Level Similarity?



The Future?

- Test the boundaries of the Complex Dynamics | Chaotic aperiodic zone. (test for randomness)
- Determine the width of Complex Dynamics zone as a function of the size of the CA Universe (lattice size, rule space size, etc...). Which are the limits where this type II zone appear.
- Study this Complex Dynamics under the notion of trajectories from Gardens of Eden to cyclic attractors.