Experiments with toy models
(ongoing project)

David Rodrigues, António Fonseca, Jorge Louçã and John Symons
Summary

- Information Theory for measuring Complex Dynamics
- Input Entropy - Mutual Information - Difference Pattern Spreading Rate
- Toy models experiments
  - Simulation Run Example!
  - Input Entropy / Standard Deviation
  - Mutual Information
  - Difference Pattern Spreading Rate.
- Generalization
Our Experiments With CAs

- **Objective**: Explore the limits and implications of some practices in the detection of Complex Behavior and Emergence

- Wolfram Rule 110 (and similar)

- Conway’s Life (B3S23) (and similar)

  - Input Entropy / Variance

  - Mutual Information between Consecutive States

  - Difference Pattern Spreading Rate
Experimental Setup

- Life board of size 120x120 in a torus configuration for boundary conditions
- Initial Random Distribution of Lit Cells (quiescent state), from 5% to 95% (19 data points)
- Run the simulation 50 times for averaging purposes for each data point over 2500 timesteps
- Headless deployment in a parallel Grid Setup.
Video Demo of the Setup
Input Entropy

- Accounts for the *uncertainty/variability* present in the histogram of transition function $\Delta$ that is applied at each timestep.

$$S^t = -\sum_{i=1}^{n} \left( \frac{Q_i^t}{n} \times \log \left( \frac{Q_i^t}{n} \right) \right)$$

- High Values of Input Entropy => Interesting Dynamics (complex?)
- Maximal Standard Deviation Zone => Interesting Dynamics.
Input Entropy
Input Entropy
Input Entropy decay

Data and Fits

\[ y = 13.11x^{-0.296} \]

Residuals
Input Entropy decay

\[ y = 13.11 \times 0.296 \]

\[ \approx -0.3 \]
Input Entropy Standard Deviation
Input Entropy Standard Deviation
Mutual Information of consecutive states

- Accounts for the information that a state as that is due to the other state

\[ I(X;Y) = H(X) - H(X \mid Y) = \sum_{k=1}^{m} \sum_{k=1}^{m} P_{X,Y} \log \left( \frac{P_{X,Y}}{P_X P_Y} \right) \]

- Consecutive states (t->t+1)

- High Values of Mutual Information => Dependence of one state from the other

- Maximal Zone => Interesting Dynamics (More information is carried through the simulation)
Mutual Information
Mutual Information
Difference Pattern Spreading Rate

* Measures the rate of “divergence” of two runs when only a small fraction of initial automata start in different states.

\[ d(X(t), Y(t)) = \sum_{i,j} |X_{ij}(t) - Y_{ij}(t)| \]

\[ \gamma = \frac{d(X(t), Y(t)) - d(X(t_0), Y(t_0))}{t - t_0} \]

* Accounts for the dependency of initial conditions.

* High Values of DPSR => 2 Runs diverge at the maximal rate...

* Maximal Zone => Interesting Dynamics
Difference Pattern Spreading Rate
Difference Pattern Spreading Rate
1. Stable configurations. These include oscillators and homogeneous configurations as when all cells die.

2. Complex Dynamics. The system presents high values of Input Entropy and Variance, Mutual Information of consecutive states and in Difference Pattern Spreading Rate.

3. The system is undistinguishable from a random system generator in the timeframe of analysis.
Type 2 - Complex Dynamics.

- Have Both High values of Input Entropy and Standard Deviation
- The DPSR test will reveal high values of DPSR
- Mutual Information is carried along the simulation
- Combination of these properties.
Wolfram Class Classification?

- Is this classification similar to Wolframs (1984) for Sets of CAs?
  - Class I - Homogenous state
  - Class II - Simple periodic structures
  - Class III - Chaotic aperiodic patterns
  - Class IV - Complex patterns of localized structures
Multi-Level Similarity?

CAS

Class I
Homogenous

Stable 1
Complex Dynamics 2
Chaotic 3

(...)

Class II
Oscillators

Stable 1
Complex Dynamics 2
Chaotic 3

(...)

Class IV
Complex Local
Structures

Stable 1
Complex Dynamics 2
Chaotic 3

(...)

Class III
Chaotic

Stable 1
Complex Dynamics 2
Chaotic 3
The Future?

- Test the boundaries of the Complex Dynamics | Chaotic aperiodic zone. (test for randomness)
- Determine the width of Complex Dynamics zone as a function of the size of the CA Universe (lattice size, rule space size, etc...). Which are the limits where this type II zone appear.
- Study this Complex Dynamics under the notion of trajectories from Gardens of Eden to cyclic attractors.